WAVEFORM LOGIC: FORMAL SPECIFICATION

A Novel Mathematical System over Oscillatory Objects

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1. Waveform Numbers ($\mathbb{W}$)

Def 1 (Waveform Number). *A* ***waveform number*** *is a quadruple* $w = \langle f, A, \varphi, \sigma\rangle$ *where* $f \in \mathbb{R}^+$ *is frequency,* $A \in \mathbb{R}$ *is amplitude,* $\varphi \in [0, 2\pi)$ *is phase, and* $\sigma : \mathbb{R} \to [-1, 1]$ *is a* ***shape function*** *satisfying* $\sigma(t + 1) = \sigma(t)$*. The space of all waveform numbers is* $\mathbb{W}$*.*

Def 2 (Instantaneous Value). *The* ***realisation*** *of* w *at time* t *is* $\hat{w}(t) = A \cdot \sigma(ft + \varphi)$*.*

Key distinction from $\mathbb{R}$**:** Two waveform numbers w_1, w_2 may satisfy $\hat{w}_1(t_0) = \hat{w}_2(t_0)$ at some instant yet $w_1 \neq w_2$. Equality in $\mathbb{W}$ requires identity across *all four* parameters. Numbers carry *history and behaviour*, not just magnitude.

2. Novel Structures

2.1 2.1 The Oscillation Product ($\circledcirc$)

Unlike standard multiplication or convolution, the **oscillation product** generates a *new* waveform number whose shape is itself a function of the input shapes:

Ax 1 (Oscillation Product). *For* $w_1 = \langle f_1, A_1, \varphi_1, \sigma_1\rangle$ *and* $w_2 = \langle f_2, A_2, \varphi_2, \sigma_2\rangle$*:*

$$w_1 \circledcirc w_2 \;:=\; \langle f_1 f_2,\, A_1 A_2,\, \varphi_1 + \varphi_2,\, \sigma_1 \circledast \sigma_2\rangle$$

where $(\sigma_1 \circledast \sigma_2)(t) := \sigma_1\big(\sigma_2(t) \cdot t\big)$ *is* ***shape composition****:* σ_2 *modulates the argument of* σ_1*.*

This is not ring multiplication. It is *non-associative* in general: $(w_1 \circledcirc w_2) \circledcirc w_3 \neq w_1 \circledcirc (w_2 \circledcirc w_3)$ because shape composition is path-dependent. This is by design: it encodes the physical fact that the order of wave interactions matters.

2.2 2.2 The Coherence Measure (γ)

Def 3 (Coherence). *For* $w_1, w_2 \in \mathbb{W}$*, define the* ***coherence*** *as:*

$$\gamma(w_1, w_2) := \lim_{T\to\infty} \frac{1}{T}\int_0^T \hat{w}_1(t)\, \hat{w}_2(t)\, dt$$

Coherence is a **scalar extracted from a pair of waveform numbers**. Unlike an inner product, γ is sensitive to shape: $\gamma(w, w) \neq A^2$ in general for non-sinusoidal σ. It defines a *non-Euclidean* geometry on $\mathbb{W}$ where distance depends on waveform morphology, not just magnitude.

2.3 2.3 The Drift Operator ($\rightsquigarrow$)

Def 4 (Drift). *The* ***drift*** *of* $w = \langle f, A, \varphi, \sigma\rangle$ *by parameter* $\delta \in \mathbb{R}$ *is:*

$$w^{\rightsquigarrow\delta} := \langle f + \delta A,\, A \cdot e^{-|\delta|},\;\; \varphi + \delta f\;,\, \sigma^{(|\delta|)}\rangle$$

where $\sigma^{(\alpha)}(t) := (1 - \alpha)\sigma(t) + \alpha\,\sigma(t)^3$ *for* $\alpha \in [0, 1]$*, clamped.*

Drift has **no classical analogue**. It simultaneously shifts frequency (proportional to amplitude), decays amplitude (exponentially in δ), rotates phase (proportional to frequency), and *deforms* the shape function toward its own cube. Repeated drift causes any waveform to **converge toward a square wave**—a fixed point attractor in shape space. This models dissipation and crystallisation simultaneously.

2.4 2.4 Waveform Entanglement ($\bowtie$)

Def 5 (Entangled Pair). *Two waveform numbers* w_1, w_2 *are* ***entangled****, written* $w_1 \bowtie w_2$*, if there exists a* ***constraint function*** $\Phi : \mathbb{W} \to \mathbb{W}$ *such that any operation on* w_1 *induces* Φ *on* w_2*:*

$$\forall\, T : \mathbb{W} \to \mathbb{W}, \quad T(w_1) \implies w_2 \mapsto \Phi \circ T(w_2)$$

This is **not** quantum entanglement—it is a structural dependency within $\mathbb{W}$ itself. Entangled numbers form *irreducible pairs*: you cannot manipulate one without transforming the other. This introduces a fundamentally **relational** arithmetic where isolated computation on single numbers is not always possible.

2.5 2.5 Collapse ($\lfloor\cdot\rceil$)

Def 6 (Collapse). *The* ***collapse*** *of* $w \in \mathbb{W}$ *is the map* $\lfloor\cdot\rceil : \mathbb{W} \to \mathbb{R}$ *defined by:*

$$\lfloor w\rceil := A \cdot \gamma(w, e_0)$$

where $e_0 = \langle 1, 1, 0, \sin\rangle$ *is the* ***unit oscillator****.*

Collapse projects a waveform number onto $\mathbb{R}$, extracting a single real value. Information is lost: many distinct elements of $\mathbb{W}$ may collapse to the same real number. This is the formal bridge to conventional mathematics, and it is *irreversible by design*—paralleling measurement in physics.

3. Structural Properties

Prop 1 ($\mathbb{W}$ is not a field). $(\mathbb{W}, +, \circledcirc)$ *does not form a field. The oscillation product is non-associative and admits no general inverse.* $\mathbb{W}$ *is a* ***non-associative algebra with drift****—a structure without precedent in standard algebra.*

Prop 2 (Drift Fixed Point). *For any* $w \in \mathbb{W}$ *with* $A > 0$*, repeated application of drift with* $\delta > 0$ *converges:* $\lim_{n\to\infty}(w^{\rightsquigarrow\delta})^{\rightsquigarrow\delta}\cdots = \langle f^*, 0, *, \mathrm{sq}\rangle$ *where* sq *is the square wave. Every waveform* ***dies toward the same shape****.*

Prop 3 (Collapse Non-injectivity). $\exists\; w_1 \neq w_2 \in \mathbb{W}$ *such that* $\lfloor w_1\rceil = \lfloor w_2\rceil$*. Conventional arithmetic operates on the* ***collapsed shadow*** *of a richer structure.*

4. Axiom System (Complete)

- **A1.** Every element of $\mathbb{W}$ is a quadruple $\langle f, A, \varphi, \sigma\rangle$.
- **A2.** Superposition is commutative and associative with identity $\langle f, 0, 0, \sigma\rangle$.
- **A3.** Phase inversion $\varphi \mapsto \varphi + \pi$ yields the additive inverse.
- **A4.** The oscillation product $\circledcirc$ is commutative but *not* associative.
- **A5.** $\circledcirc$ distributes over $+$ from the left: $w_1 \circledcirc (w_2 + w_3) = (w_1 \circledcirc w_2) + (w_1 \circledcirc w_3)$.
- **A6.** Drift is a one-parameter semigroup: $w^{\rightsquigarrow 0} = w$, $(w^{\rightsquigarrow\alpha})^{\rightsquigarrow\beta} = w^{\rightsquigarrow(\alpha+\beta)}$ up to shape deformation.
- **A7.** Entanglement is symmetric but not transitive.
- **A8.** Collapse is a surjective homomorphism from $(\mathbb{W}, +)$ to $(\mathbb{R}, +)$.
- **A9.** **No element of $\mathbb{W}$ is fully determined by its collapse.**

5. Why This Is New

Concept	Distinction from existing maths
Oscillation product	Non-associative multiplication via shape composition; no analogue in ring theory
Drift	Simultaneous coupled deformation across all four parameters; not a group action
Coherence geometry	Distance depends on shape morphology, not norm; non-Euclidean in a novel sense
Entanglement	Structural dependency making isolated arithmetic impossible; no classical parallel
Collapse	Irreversible many-to-one projection; real numbers as lossy compression of $\mathbb{W}$